Lecture #1

- Parser software: CVX (in MATLAB) Software
  CVXPY (in Python)
  Convex.jl (in Julia) Programming language

- Install Jupyter Notebook

- Math notations: x, $\underline{x}$, X, $\mathcal{X}$
  (scalar) (vector) (matrix) (set)

- Optimization Problem:

$$\boxed{\begin{array}{ll} \text{minimize} & f_0(x) \\ \text{subject to} & f_i(\underline{x}) \leq b_i \end{array}} \boxed{, \quad i=1,\ldots,m}$$

- Optimization var. $\underline{x} = (x_1, \ldots, x_n)^T$

objective function $f_0 : \mathbb{R}^n \mapsto \mathbb{R}$

constraint functions $f_i : \mathbb{R}^n \mapsto \mathbb{R}$, $b_i \in \mathbb{R}$

- Optimal Solⁿ of the problem.

$$\underline{x}^* = (x_1^*, \ldots, x_n^*)$$

- Feasible set : $\mathcal{Z} = \{ \underline{z} \in \mathbb{R}^n \mid f_i(\underline{z}) \le b_i \}$

set

equiv. | $\text{minimize } f_0(\underline{x})$, $\underline{x} \in \mathcal{Z}$

$$\text{if } \forall\, \underline{z} \in \mathcal{Z},\ f_0(\underline{z}) \ge f_0(\underline{x}^*)$$

Historically optimization problems have been classified

Linear (All of the fⁿs $f_0, f_1, \ldots, f_m$ are linear) vs. Nonlinear (Any one of the functions $f_0, f_1, \ldots, f_m$ is Nonlinear)

$$\text{minimize } \underline{c}^T \underline{x} =: \langle \underline{c}, \underline{x} \rangle = f_0(\underline{x})$$

$$\underline{a}_i^T \underline{x} \le b_i, \quad i = 1, \ldots, m$$

Linear optimization problem:

$$\underset{\underline{x}}{\text{minimize}} \quad c_1x_1 + c_2x_2 + \dots + c_nx_n \iff \boxed{\begin{array}{ll} \text{minimize} & \underline{c}^T\underline{x} \\ \text{s.t.} & \underline{a}_i^T\underline{x} \le b_i, \\ & i = 1, \dots, m \end{array}}$$

$$\text{s.t.} \quad a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \le b_1$$

$$\vdots \qquad\qquad\qquad \vdots$$

$$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n \le b_m$$

$$\boxed{\begin{array}{l} \text{minimize} \quad \underline{c}^T\underline{x} \\ \text{s.t.} \; A_{m\times n}\,\underline{x}_{n\times 1} \le \underline{b}_{m\times 1} \end{array}}$$

Convex Optimization Problems have $f_0, f_1, \dots, f_m$ convex functions

<u>Linear function</u>

$$f(\alpha\underline{x} + \beta\underline{y}) = \alpha f(\underline{x}) + \beta f(\underline{y})$$

Then $f(\cdot)$ is <u>linear</u>

$\forall\ \underline{x}, \underline{y} \in \mathbb{R}^n$

$\forall\ \alpha, \beta \in \mathbb{R}$ ✓

<u>Convex function</u>

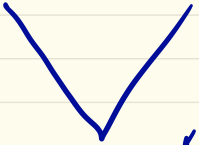
$$f(\alpha\underline{x} + \beta\underline{y}) \le \alpha f(\underline{x}) + \beta f(\underline{y})$$

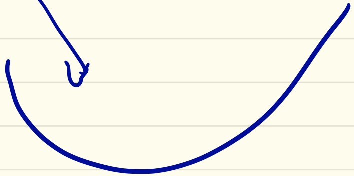
$\forall\ \underline{x}, \underline{y} \in \mathbb{R}^n$

and $\alpha, \beta \ge 0$ ✓

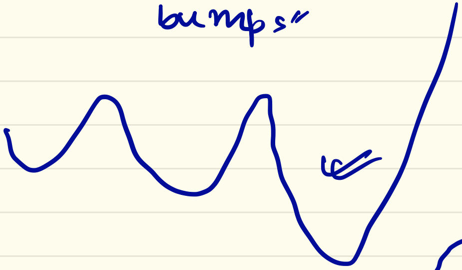
s.t. $\alpha + \beta = 1$

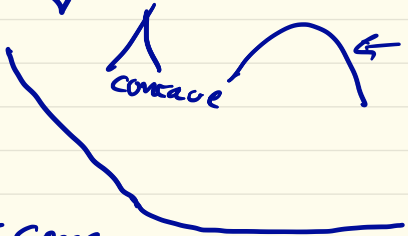
Geometric intuition: Convex functions are "bowl shaped"





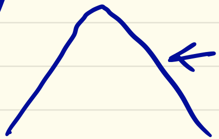
Non-convex = "multiple bumps"





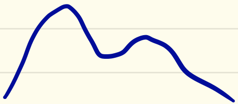
concave

concave



concave

Convex $\neq$ easy



$\min f_0(A)$

s.t. $A \in \mathcal{A}$

$\mathcal{A}$ = set of co-positive matrices

$$= \{ A = A^T \mid x^T A x \geq 0 \text{ for all } \cancel{x \geq 0} \}$$