Convex optimization problems

Lecture #8

$$\min_{x \in \mathcal{X}} f(x) \text{ is convex iff}$$

$f(\cdot)$ is a convex $f^{\underline{n}}$ of x

& $\mathcal{X}$ is a convex set.

→ LP (linear Program)

$c^T x$

Both f is linear & $\mathcal{X}$ is defined as intersection of halfspaces & hyperplanes

Any LP:

$$\begin{aligned} \min \quad & c^T x \\ \text{s.t.} \quad & A\underline{x} \leq b \end{aligned}$$

minimize linear $f^{\underline{n}}$ over polyhedron

→ QP (Quadratic Programs)

$$f(\underline{x}) = \frac{1}{2}\underline{x}^T A \underline{x} + \underline{b}^T \underline{x} + c \qquad A \in \mathbb{S}^n_+$$

$$\mathcal{X} : \quad P\underline{x} \preceq \underline{q}$$

Minimize convex quadratic f^n over polyhedron

→ QCQP (Quadratically constrained quadratic Program)

$$f(\underline{x}) = \frac{1}{2}\underline{x}^T A \underline{x} + \underline{b}^T \underline{x} + c$$

$$\mathcal{X} : \frac{1}{2}\underline{x}^T M_i \underline{x} + \underline{n}_i^T \underline{x} + r_i \leq 0, \quad i = 1, \dots, m \qquad M_i \in \mathbb{S}^n_{++}$$

$$P\underline{x} \preceq \underline{q}$$

Minimize convex quadratic f^n over the intersection of m ellipsoids and a polyhedron

→ SOCP (Second order cone program)

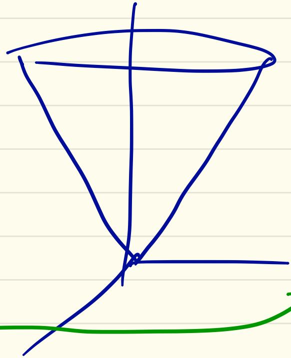
$$\min \; \underline{f}^T \underline{x}$$

s.t. $\|A_i \underline{x} + \underline{b}_i\|_2 \le \underline{c}_i^T \underline{x} + d_i, \quad i = 1, \dots, m$

$F\underline{x} \le \underline{g}$, $\quad A_i \in \mathbb{R}^{n_i \times n}$

$F \in \mathbb{R}^{p \times n}$

$\underbrace{(A_i \underline{x} + b_i, \underline{c}_i^T \underline{x} + d_i)}$ define second order cone in $\mathbb{R}^{n_i+1}$



If $n_i = 0$, then LP

If $c_i = 0$, then QCQP

Minimize linear f^n over intersection of polyhedron & cone

→ SDP (Semi-definite program)

$$\min_{X \in S_+^n} \; \mathrm{tr}(C^T X) \quad \text{s.t. } \mathrm{tr}(A_k^T X) \le b_k, \quad k = 1, \dots, m$$

x: linear matrix inequality

$$LP \subseteq QP \subseteq QCQP \subseteq SOCP \subseteq SDP$$